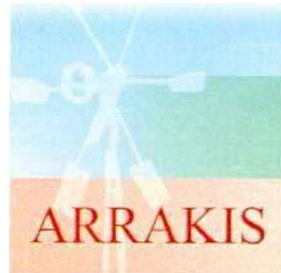


CONTROL AND SAFETY SYSTEMS OF WATER PUMPING WINDMILLS BASICS

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October 1991



Latest update: Dec 2012

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1 INTRODUCTION

Control and safety systems form an essential part of a water pumping windmill, without these the mill would not survive. There are two modules in this course on this subject: the basics, presented in this module, and the advanced module of A. Kragten.

In this module the terms control and safety are explained first, followed by a historical overview of control and safety systems. The current mechanisms to obtain control and safety actions are highlighted.

Two comparisons are made. Firstly the actual control and safety characteristics are compared with the ideal characteristics. Secondly the actual characteristics are compared with theoretical predictions.

The theory itself, at least the base line, is presented in an annex for the interested reader. Further details are discussed in the advanced module of Kragten.

For convenience every chapter starts with a short description of its contents.

2 CONTROL AND SAFETY

Control and safety are two different concepts, though they are related to some extent. The two concepts are discussed in this chapter.

2.1 Control

A windmill will have to face all weather conditions, from stills to storms. During normal weather conditions (low and medium wind speeds) the control mechanism will have to secure an efficient and smooth operation of the windmill. During storms safety is more important. This section concentrates on control, the next section on safety.

In what way is efficient operation of the windpump realised? Most windpumps today use yawing of the rotor as the means to secure the control objective. For low winds the control mechanism will direct the rotor such that it is facing the wind; thus optimum use can be made of the wind. For medium wind speeds the control mechanism will turn the rotor partly out of the wind. Thus the generated power, rotor speed and forces will be kept in hand. Figure 2.1 shows the rotor of a windmill in a bird's-eye view.

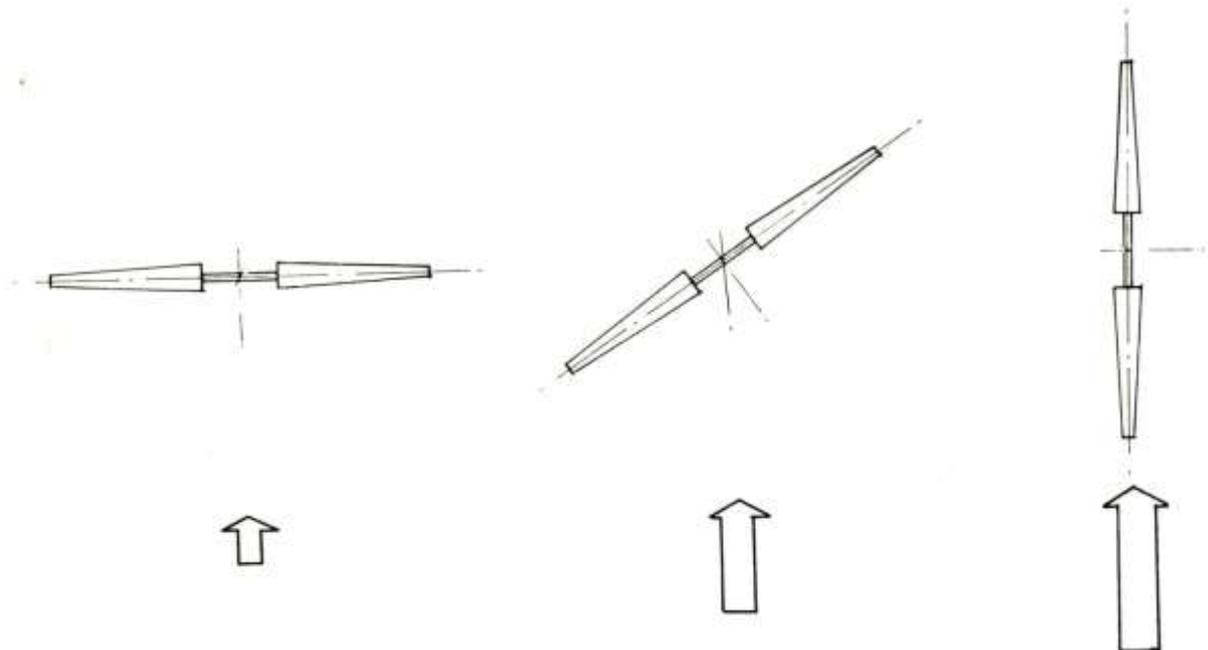


Figure 2.1 Control and safety action

2.2 Safety

If the windmill experiences a storm the safety mechanism has to prevent the mill from being blown to pieces.

The safety mechanism uses yawing of the rotor too in most modern windmills; the rotor will practically be parallel to the wind in case of storms, see figure 2.1.

Apart from the automatic safety mechanism most windmills have a manually operated device (a lever or winch) to turn the rotor out of the wind. Thus the windmill may be put in yaw when the miller is expecting a severe storm.

Some windmills have, in addition to the yawing mechanism, an automatic brake that is activated when the mill is in yaw (e.g. Dempster). In other windmills a stick may be pushed in the rotor (of course after it has stopped). This will prevent the rotor from turning in case of sudden wind shifts.

2.3 Comparison between control and safety

Though control and safety are two different actions they both use the same principle in today's windmills; the yawing of the rotor. The goals are, shortly;

-control action: efficient operation

-safety action: limitation of forces.

3 HISTORIC DEVELOPMENT

Windmills have been used for many centuries. The control and safety mechanisms have evolved over the years. This chapter gives an overview.

3.1 Windmills without automatic control

In the early days a common type of windmill was the 'European' windmill. A four bladed wheel was mounted on a swiveling cap set atop a permanent stationary tower.

The mills were used for grinding grain into flour, wood sawing and, sometimes, water pumping (mostly drainage).

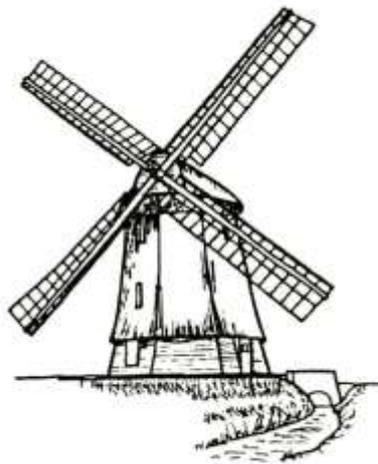


Figure 3.1 European windmill

The European windmills required constant attention of the miller, in fact the miller often lived in the tower. It was the miller that carried out the required control and safety measures. If the miller anticipated periods with high winds he partly removed the sails from the blades. Thus he was able to control the speed. If a storm was coming he fully removed the sails as a safety measure.

The principle of (partly) removing sails was also used for other types of windmills, like the Cretan windmill.

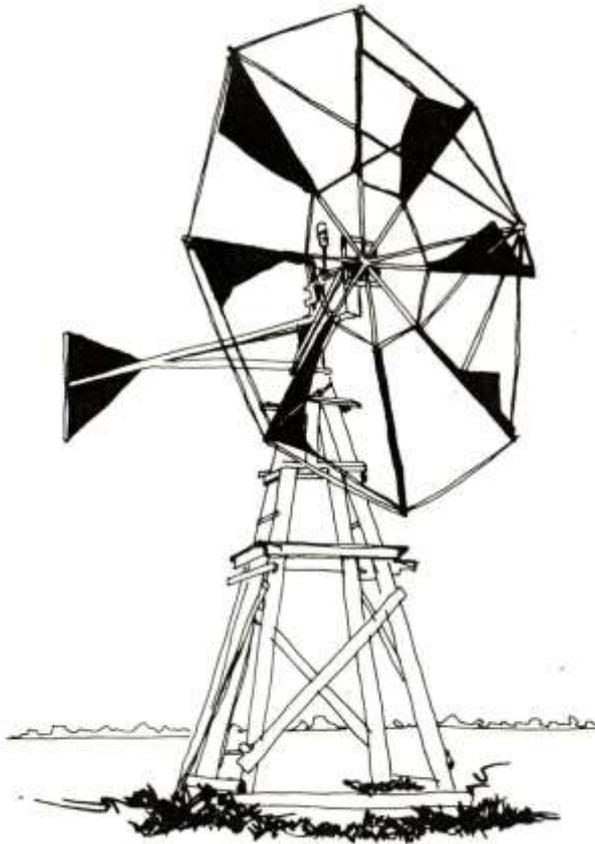


Figure 3.2 The Cretan windmill

Sometimes a bottle was fixed to the tip of a blade of a Cretan windmill. The bottle started to 'sing' when the rotor speed became high, alarming the miller to reef the sails.

3.2 Windmills with automatic control

Though this way of securing control and safety was generally sufficient it was not very efficient as it required constant attention. The availability of a water pumping windmill that would automatically control and regulate its operation without human attention became urgent during the cultivation of vast plains in, a.o., the United States, during the second half of the 19th century. T. Lindsay Baker describes the development in the United States in his book 'A field guide to American windmills' [1].

3.2.1 Pitch control

Lindsay Baker writes: 'The first successful self-governing American windmill was invented in 1854 ... by Daniel Halladay'.

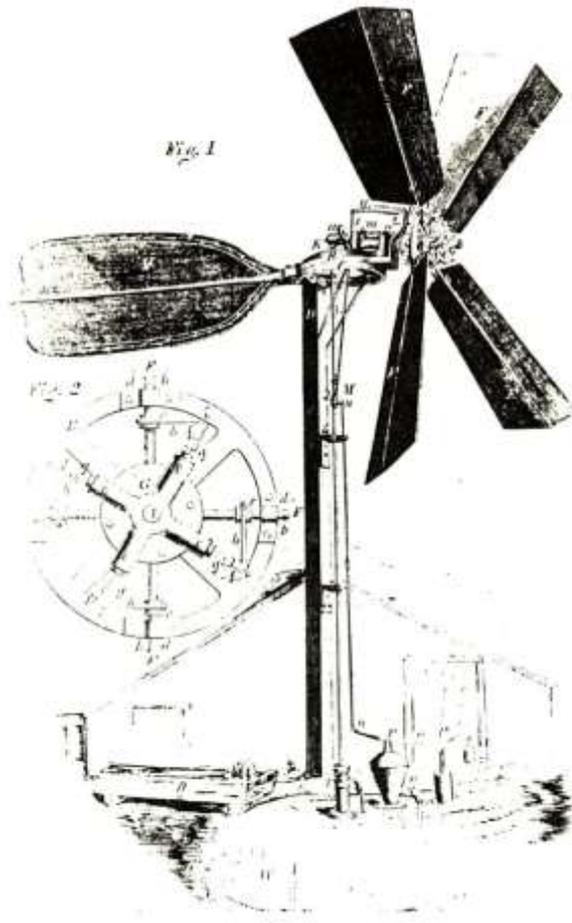


Figure 3.3 Early windmill with pitch control

In this windmill the rigid vane always kept the wheel facing the wind like a weather vane on top of a barn. In light winds the blades were placed at such an angle as to derive the greatest power from it (the control action). As the wind velocities increased and the wheel spun faster, a centrifugal governor changed the pitch of the blades so that they became less efficient, thus controlling their speed (the safety action). The blades were called 'self-furling sails'. The principle of pitch control is still in use for modern electricity generating wind turbines.

The windmill became a large success. The windmill was not only used by the farmers but, e.g., also by the railway companies who used it to fill the water reservoirs for the passing steam locomotives.

3.2.2 Sectional wheel control

Another type of control mechanism appeared somewhat later than the Halladay mill: the sectional wheel system.

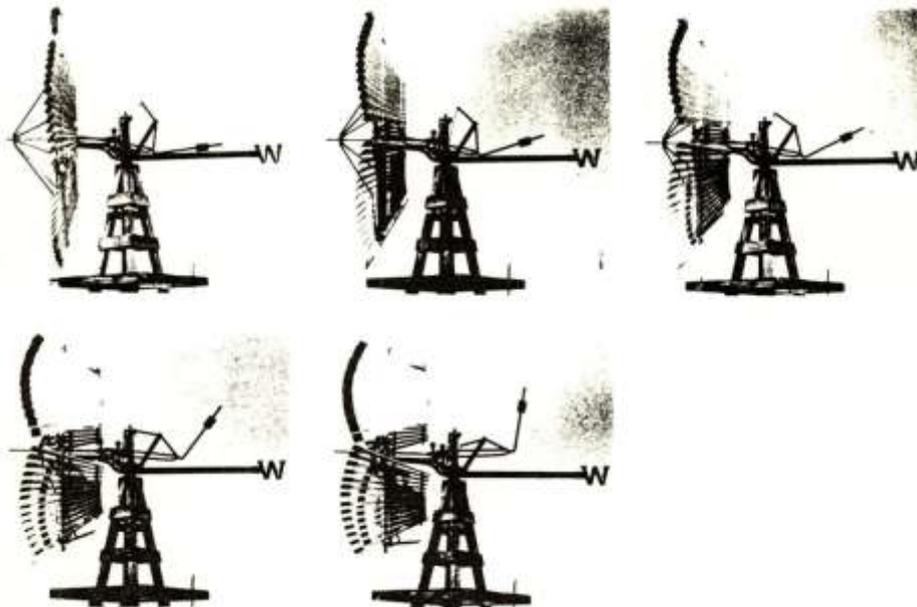


Figure 3.4 The sectional wheel control

Sections of blades were mounted such that the individual sections could pivot to regulate the amount of their surface area exposed to the wind (it looked more or less like the (un)folding of an umbrella). Both wind pressure and rotor speed influenced the pivoting of the sections. By gradually turning the sections the control and safety actions could be realised. Note that the rotor turns on the lee side of the tower (down wind), no vanes are required.

3.2.3 Yaw control

A third development, soon becoming the leading arrangement, was the yaw control mechanism. It was introduced around 1866. The windmill head with rotor is placed on a turn table. The head automatically turns out of the wind for larger wind speeds and automatically sways back in the wind when the wind speed drops. Thus the wind speed component perpendicular to the rotor is less than the actual wind speed in case of high winds (see figure 3.5). This will keep the rotor speed and corresponding forces within acceptable limits, still providing maximum power for small and medium wind speeds (see also chapter 2, especially figure 2.1).

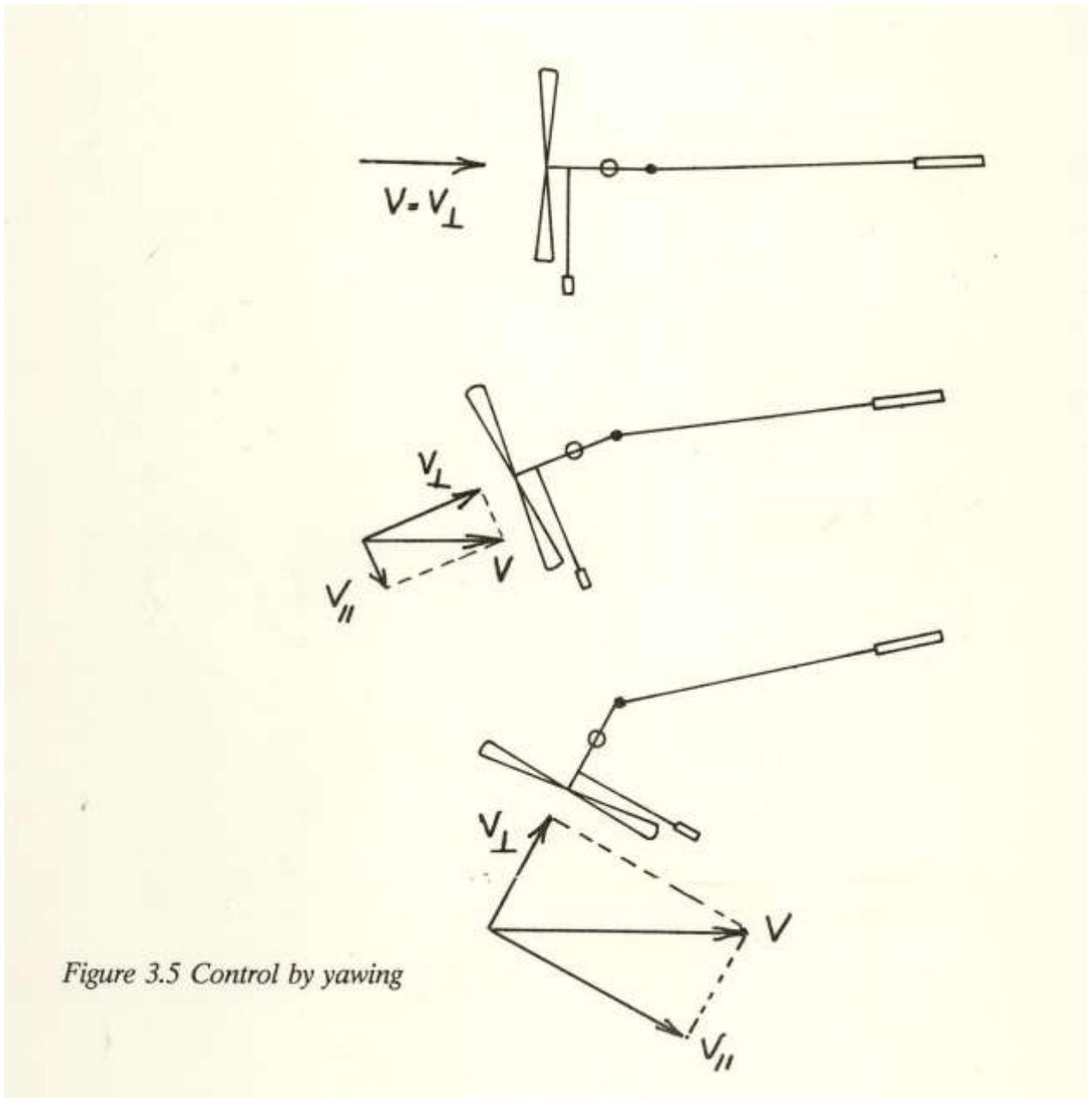


Figure 3.5 Control by yawing

Figure 3.5 Control by yawing

4 THE IDEAL CONTROL AND SAFETY MECHANISM

It appears that the 'ideal' control and safety characteristic can be formulated on the basis of the control and safety definitions. This chapter shows this, making use of one small formula.

For almost all current windmills the control and safety actions are realised by yawing of the rotor. The yaw angle is the angle δ between the rotor shaft and the wind speed. For each wind speed there is an 'ideal' yaw angle, that is the angle where the wind pump provides maximum power (control action) without exceeding the rotor speed (safety). The relation between wind speed and ideal yaw angle is established in this chapter.

4.1 Low and medium wind speeds

For the low and medium wind speeds the rotor should catch as much wind energy as possible. Therefore the rotor must face the wind perpendicularly, see figure 4.1a. The ideal yaw angle δ is zero, see figure 4.1 and 4.2.

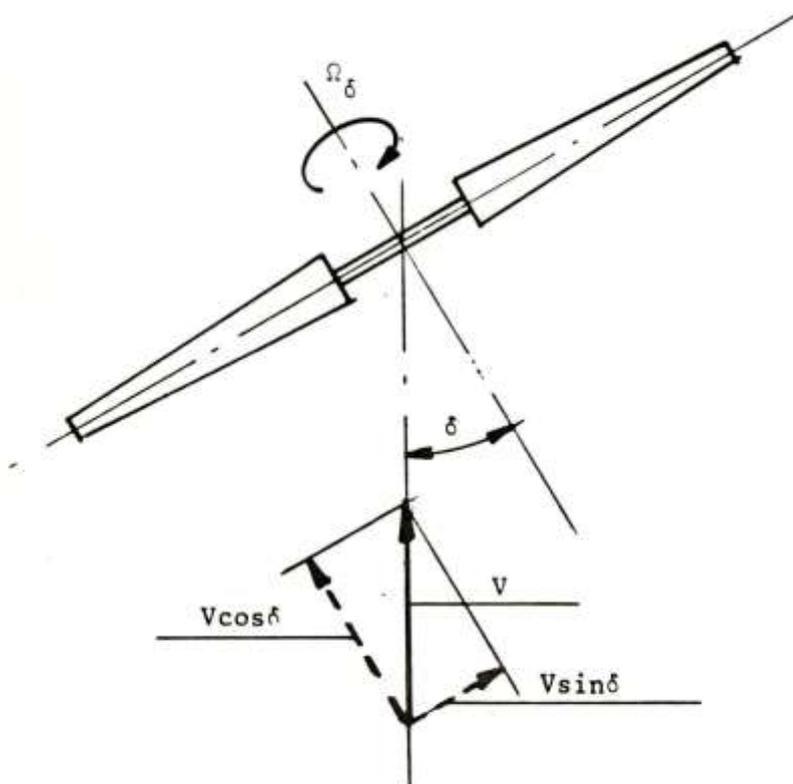


Figure 4.1 Definition of the yaw angle δ

When the wind speed has reached the so-called 'rated wind speed' V_r , the rotor speed, forces and power have reached their admissible maximum, they must not become any larger (to be precise: they may not be larger for longer periods. For short periods they may exceed the rated value to some extent as this forms no threat to the windmill).

All in all we may conclude that the rotor has to face the wind perpendicularly for wind speeds up to the rated wind speed to have full effect from the wind. In mathematical terms: the ideal yaw angle δ will be zero degrees up to the rated wind speed V_r .

For water pumping windmills the rated windspeed generally amounts to six to nine meters per second (corresponding to Beaufort scales 4 to 5; moderate to fresh breeze).

4.2 Wind speeds beyond the rated wind speed

For wind speeds beyond the rated wind speed the rotor should be (partly) turned out of the wind to keep the rotor speed and forces within acceptable limits. In the ideal case the yaw angle should be such that the wind speed component perpendicular to the rotor equals the rated wind speed, being the maximum allowable wind speed that the rotor may face. The side wind component does not have any significant effect on the rotor speed.

If we call the wind speed V (m/s) and its component perpendicular to the rotor is V_r (rated), then:

$$V_r = V \cos(\delta).$$

Or, rewriting the formula:

$$\delta = \arccos\left(\frac{V_r}{V}\right)$$

Figure 4.2 shows this relation.

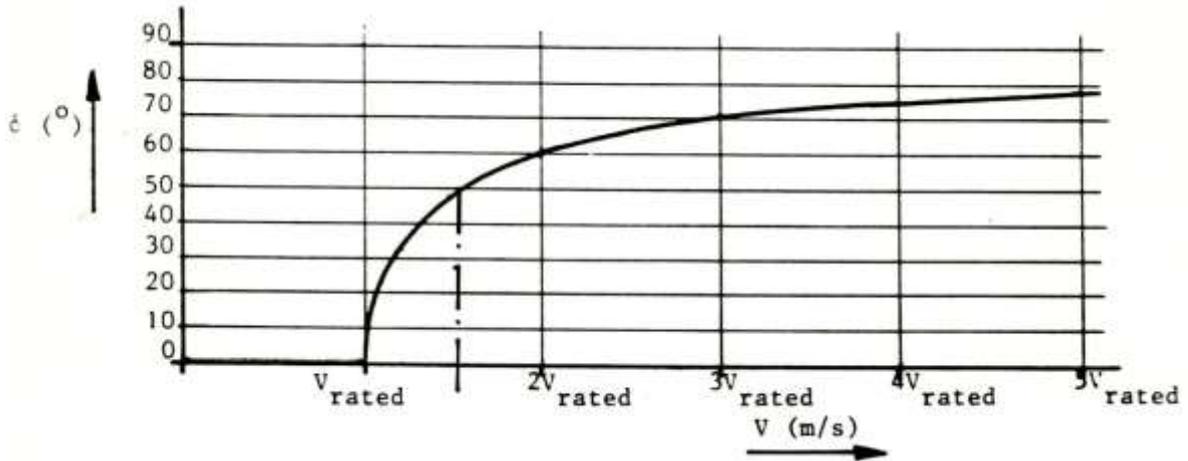


Figure 4.2 Ideal yaw angle in relation to the wind speed

Summarizing; for an ideal control system the yaw angle δ should be zero for windspeeds below the rated windspeed V_r . Above the rated windspeed the yaw angle δ should follow the indicated curve.

4.3 Safety action

For very high wind speeds, equal or larger than the so-called cut-out wind speed, the rotor should be in full yaw (yaw angle $\delta \approx 90$ degrees), just for safety reasons. The cut-out wind speed for wind pumps generally amounts around $4/3$ times the rated windspeed (hence around 8 to 12 m/s).

5 YAW CONTROL SYSTEMS

Control and safety of the water pumping windmill is done by yawing of the rotor. Some different designs are highlighted, but first the principle is discussed.

5.1 Principle

In all control and safety mechanisms two counteracting devices may be identified. The first device tries to turn the rotor out of the wind. The other device tries to turn the rotor into the wind. For every wind speed there is a balance between the two devices. The balance at higher wind speeds is reached at larger yaw angles, as it should be.

We may reformulate the control action in terms of mechanics: the first device exerts a moment on the windmill head plus rotor, trying to turn the rotor into the wind. A counteracting moment is exerted by the second device. Both moments depend on the wind speed. A balance of moments exists for every wind speed.

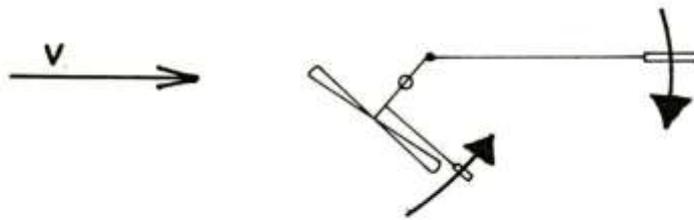


Figure 5.1 Balance of moments

Analysing the different designs in windpumps with yaw control comes down to identifying the two different devices. In the next two sections possible devices are described.

We start with devices that turn the rotor out of the wind.

5.2 turning the rotor out of the wind

There are two arrangements for turning the rotor out of the wind: the side vane and the eccentric rotor. Note that in both cases the driving force to turn the rotor is the wind force itself.

5.2.1 The side vane

A side vane is mounted to the windmill head parallel to the rotor, see figure 5.2. The wind exerts a force on the vane. This force results in a moment on the head trying to turn the head (and with it the rotor) out of the wind.

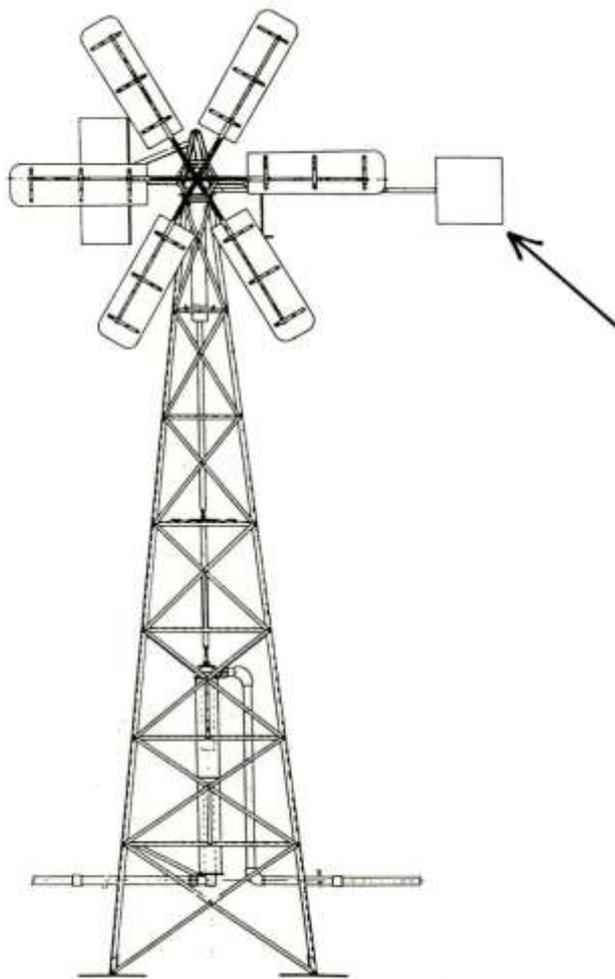


Figure 5.2 Side vane

5.2.2 Eccentric rotor

The second, and most popular method to turn the rotor out of the wind is to place the rotor eccentrically relative to the head axis. Thus the thrust force on the rotor causes a moment on the head, trying to turn the head out of the wind, see figure 5.3.

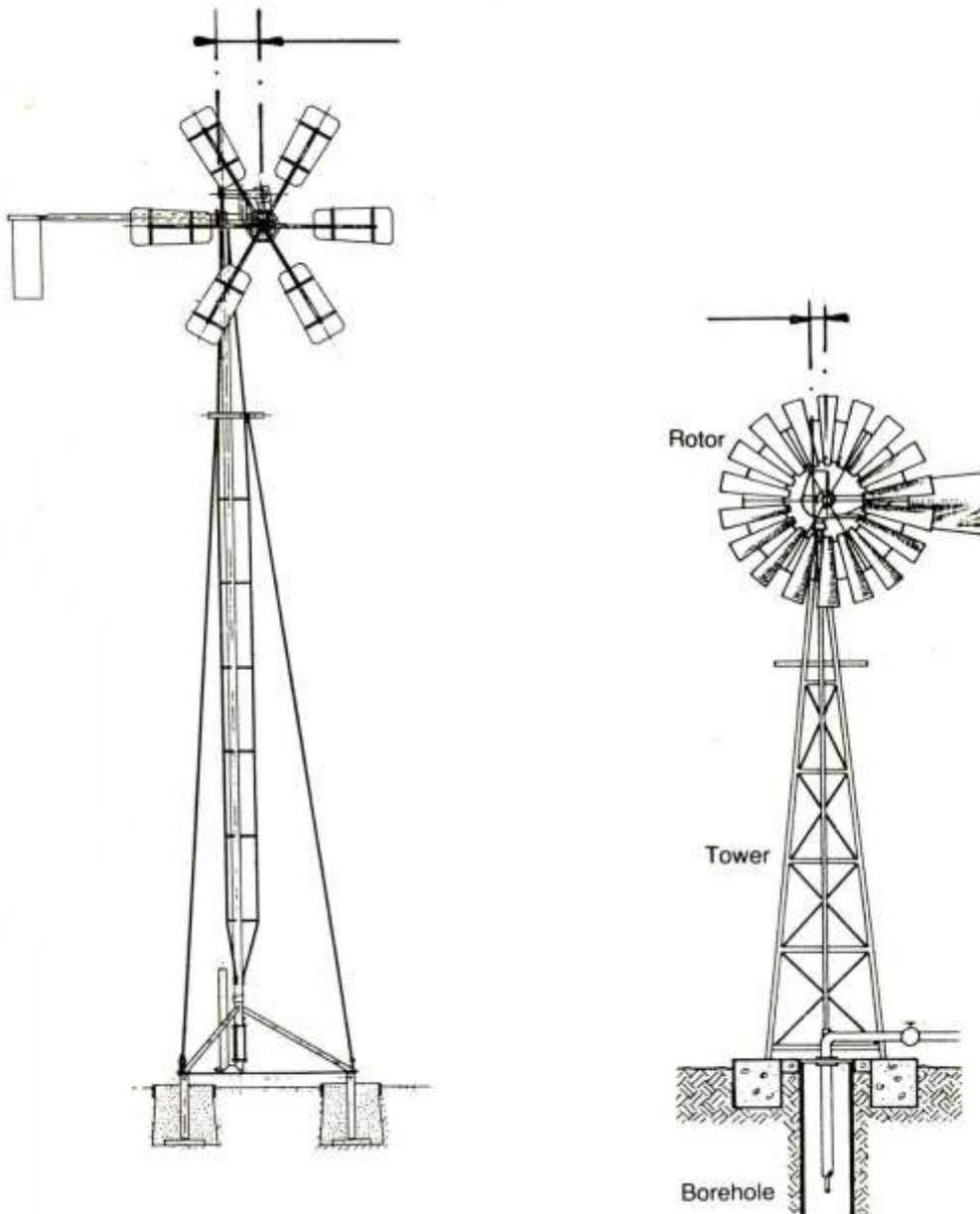


Figure 5.3 Eccentric rotor

Note that both arrangements are activated by the wind speed only. The moments become larger for higher wind speeds.

5.3 Turning the rotor into the wind

Most windmills have a main vane, also known as tail vane. Basically this main vane counterbalances the forces turning the rotor out of the wind. If the tail vane were rigidly fixed to the head the position of the rotor with respect to the wind would remain the same for every wind speed. So we see that the main vane is not rigidly fixed to the head but that it can pivot around a hinge fixed to the head. There are three different arrangements: the ecliptic main vane, the inclined hinge main vane and the hinged side vane.

Note; for some small wind pumps the tail vane is rigidly connected to the windmill head. Those small windmills (rotor diameter smaller than 1 meter) do not have a device that turns the rotor out of the wind. The rotor always faces the wind perpendicularly, whatever the wind speed. Large forces are thus generated during high wind speeds. But the small windmills will survive as they can easily be made very sturdy (being a scale effect).

5.3.1 Ecliptic main vane

The main vane pivots around a vertical hinge. If no further arrangements were made the main vane would, like a door, just pivot around the hinge if some wind pressure was applied, without exerting a moment on the head. However the main vane should exert a moment on the head, to counterbalance the moment of the side vane or eccentric rotor.

Therefore a spring is incorporated. When the main vane starts turning the spring will be extended, creating a force on the head. This force creates a moment on the head, as illustrated in figure 5.4. The system may be compared with a door in combination with a spring to keep the door shut. The spring will exert a force on the door post when you open the door.

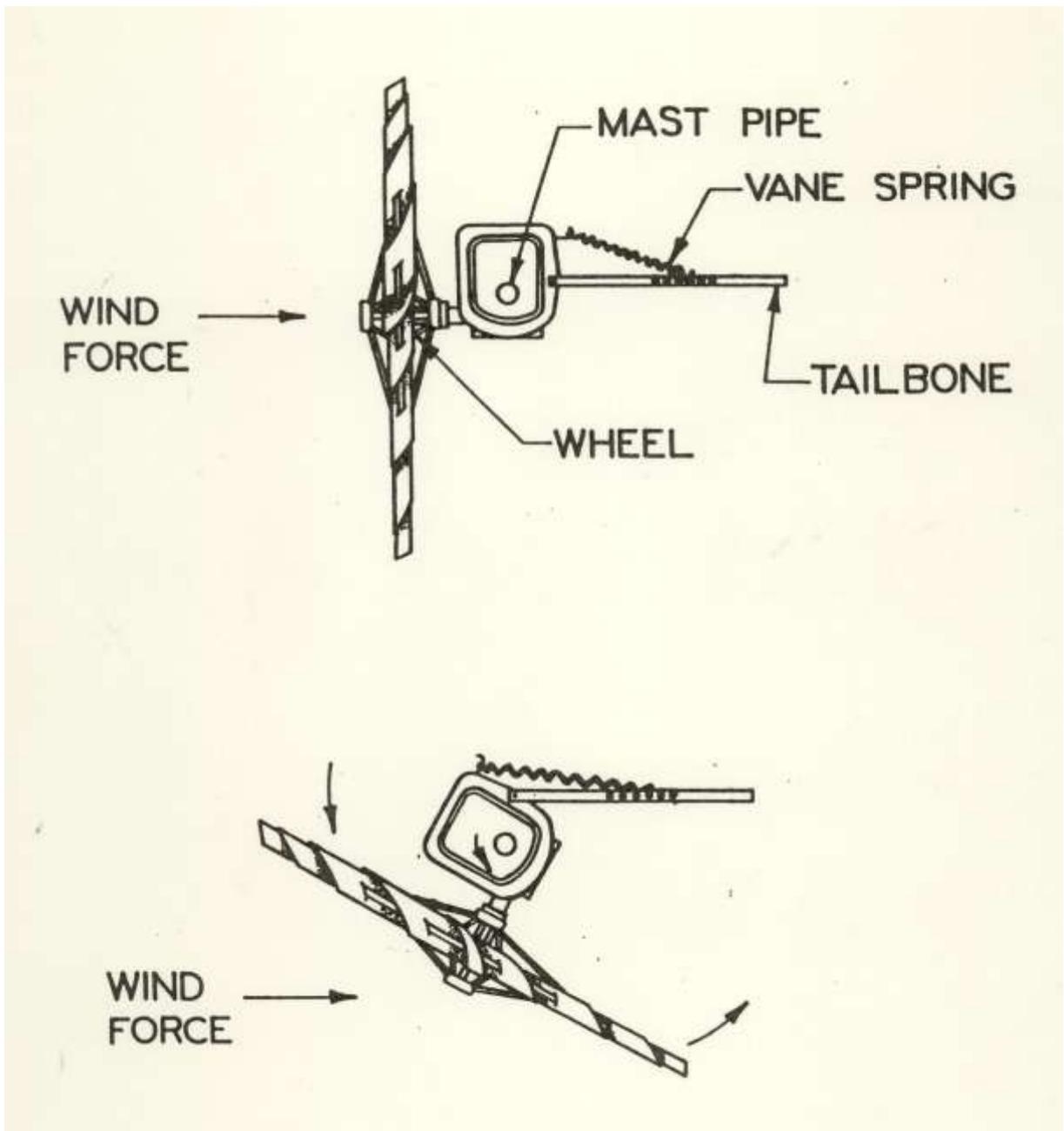
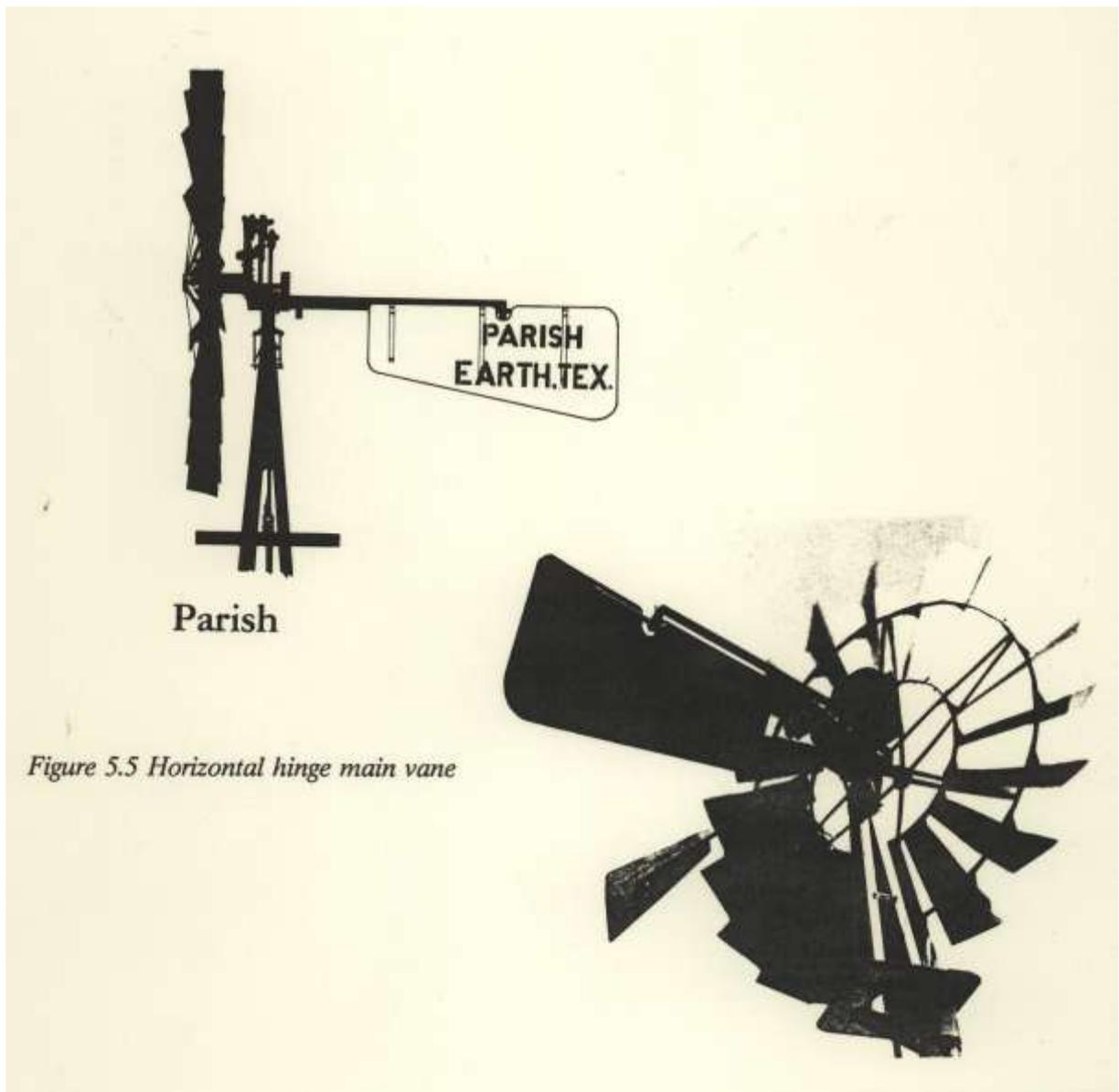


Figure 5.4 Eclipse main vane

The name stems from the windmill manufacturer 'Eclipse' who first used this system (USA, around 1866).

5.3.2 Horizontal hinge main vane

The vane pivots around a horizontal hinge. When a wind pressure is exerted on the vane blade it will be lifted against gravity, the more so if the wind speed is higher. This means that a wind force will be exerted on the vane blade, and therefore on the rotor head. Note that the weight of the blade plays an essential role in the control system (to understand this imagine the two extremes: a weightless and very heavy vane blade). Figure 5.5 gives an impression.



5.3.3 Inclined hinge main vane

As the name already suggests the main vane has an inclined hinge, generally about 10 degrees from the vertical. Would the hinge be vertical no moment would be generated on the head (compare it with a flag). As a result of the inclination the vane is able to create a moment on the head. The system resembles the horizontal hinge main vane where the weight of the vane is important. Figure 5.6 gives an impression.

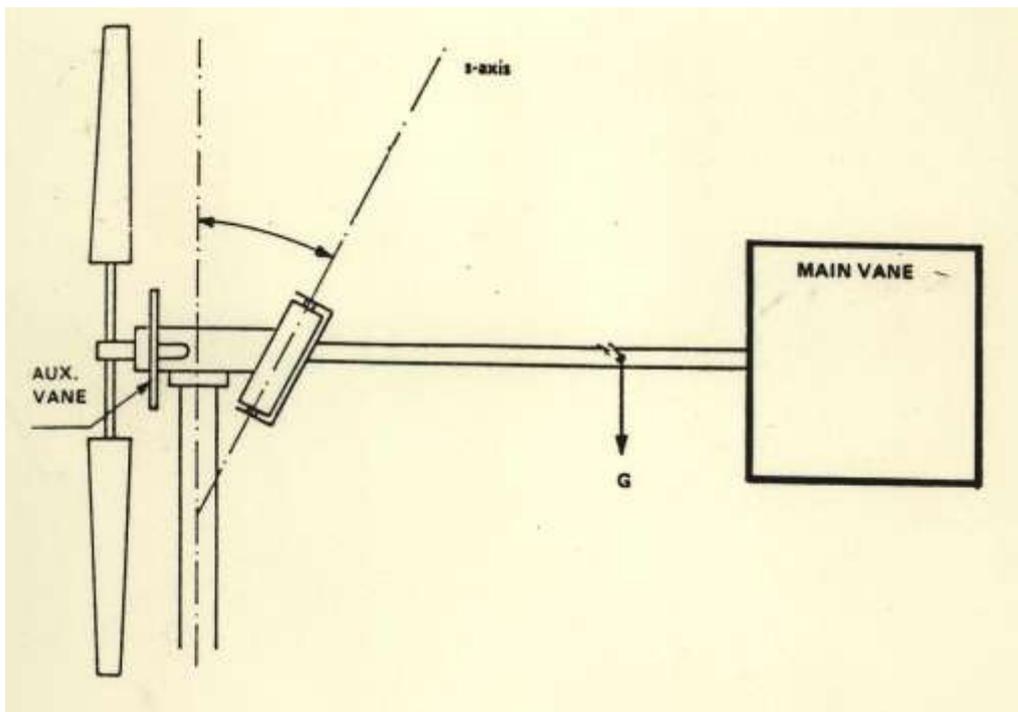


Figure 5.6 Inclined hinge main vane

5.4 Overview

In a wind pump different arrangements can be found. An overview of the combinations used in wind pump designs is given in table 5.1.

table 5.1 Overview of control systems in current designs

into the wind	out of the wind	
	side vane	eccentric rotor
ecliptic		FIASA
hor. hinge main vane *)	Parish	CWD 2000
inclined hinge vane	CWD 2740 & 5000	Southern Cross

*) The horizontal hinge main vane is often referred to in literature as the 'hinged side vane'. The latter is a confusing name as the 'side vane' turns a rotor out of the wind while the 'hinge side vane' does the opposite.

As can be seen the eccentric rotor as an arrangement to turn the rotor out of the wind is the most popular one.

6 IDEAL AND PRACTICAL CONTROL CHARACTERISTICS

A comparison between the ideal and practical control characteristics is made in this chapter.

The question is how close the actual control behaviour of a wind pump is to the ideal behaviour, as already defined in chapter 4. Unfortunately little data are available. For some wind pumps the control behaviour was measured, either in the wind tunnel or in the field. For the CWD 5000 (a 5 meter rotor diameter wind pump, with a side vane & inclined hinge main vane) windtunnel measurements on a scale model 1:10 were performed by Kieft [2]. The control characteristic is presented in figure 6.1. The characteristic is compared with the ideal characteristic. (rated wind speed 9 m/s, cut-out wind speed 12 m/s).

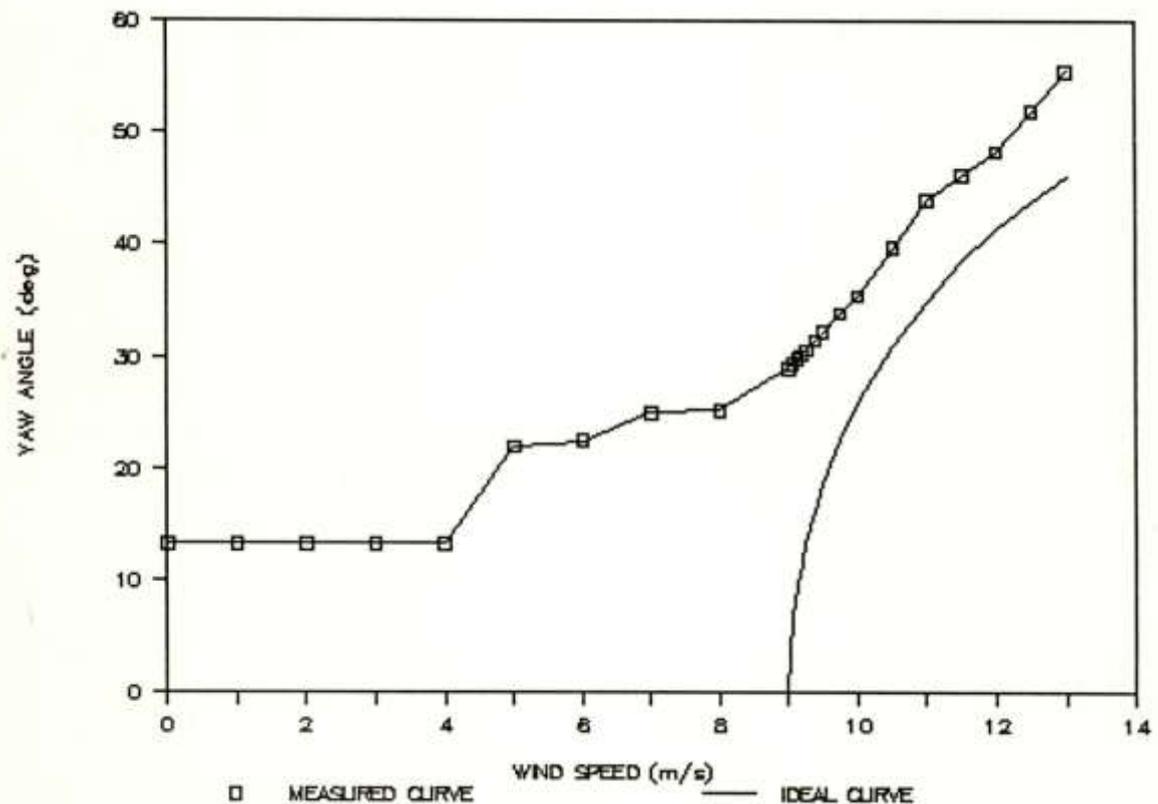


Figure 6.1 Control characteristic of CWD 5000; yaw angle versus wind speed

Figure 6.1 shows a large difference between the ideal and the practical control behaviour. For low wind speeds the rotor is already partly turned out of the wind. For example for a wind speed of 6 m/s the yaw angle is 22 degrees.

Though the difference is pronounced the effects are not dramatic; the power depends on the rotor area 'seen' by the wind; this area will still be 93 percent of the total rotor area for a yaw angle of 22 degrees.

Figure 6.1 also shows that the yaw angle is larger than required for high wind speeds. This again means a loss of power.

The fact that the control action is not optimal is the price that has to be paid for the use of a simple control system. More sophisticated control systems would give better control characteristics, but the price would be too high for water pumping windmills.

It should be noted that the control characteristic is discussed for steady wind conditions. In reality, due to sudden wind shifts etc, the yaw angle may vary around the static value. It is therefore wise to take some distance from the ideal curve (in other words; to yaw more than strictly required) such that disturbances do not immediately lead to dangerous situations.

Another way of judging the control behaviour of windmills is by looking at the rotor speed in relation to the wind speed. Figure 6.2 gives the relation between the rotor speed and the wind speed for the same CWD 5000 model of figure 6.1.

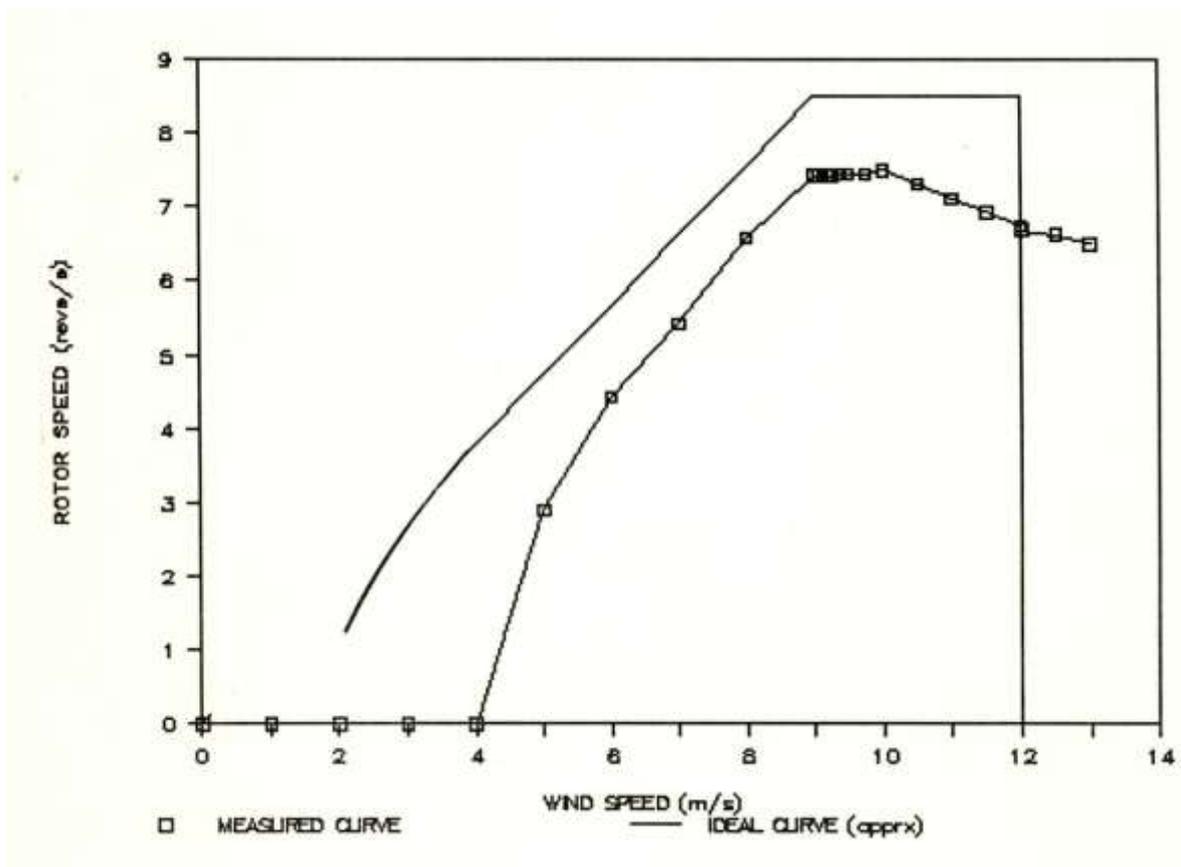


Figure 6.2 Control characteristic of CWD 5000; rotor speed versus wind speed

Note that the rotor speed has a maximum at 9 m/s wind speed, beyond 10 m/s the rotor speed decreases again. This means that the actual rated wind speed is around 9 m/s.

Other rotor speed/wind speed characteristics are given by Kragten in the advanced module 'Safety systems for water pumping windmills' of this course.

7 DYNAMIC BEHAVIOUR

Stationary situations are discussed in the previous chapters. However dynamic behaviour may occur, for instance due to sudden wind shifts or wind speed fluctuations. This chapter discusses the matter.

The control mechanism has a static control curve, being the average of a number of measurements. In reality the deviations from the average may be quite large. For example the average rotor speed may be 60 rpm at a wind speed of 5 m/s, but variations around the average between 40 and 80 rpm are not uncommon.

These variations can be seen in figure 7.1. It represents field measurements on a CWD 5000 wind pump [3].

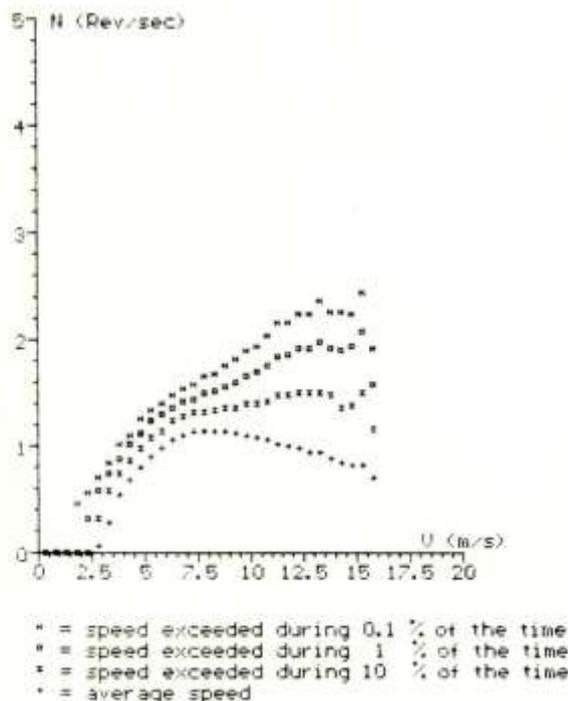


Figure 7.1 Rotor speed of CWD 5000 in relation with wind speed [3]

The lowest curve represents the average rotor speed for each wind speed bin (bin width 0.5 m/s). The other three curves represent the rotor speeds that were exceeded during 10, 1 and 0.1 percent of the time.

For example the average rotor speed for the 7 to 7.5 m/s wind speed bin was 1.15 revolutions per second, but 10 % of the time the speed was larger than 1.4 rps (1 % of the time > 1.5 rps and 0.1 % >1.6 rps).

This illustrates the significant variations from the stationary (= average) control behaviour.

8 THEORY OF CONTROL

Theory is compared with measured control behaviour. The theory itself is highlighted in annex A.

Of course theories have been developed to predict the control behaviour starting with the prediction of the stationary control behaviour. An extensive description of the theory is given in the advanced module on safety systems by Kragten. A general description of theoretical models is given in annex A. In this chapter we only compare the predictions of a theoretical model with the measurements.

In the comparison we look at the rotor speed curves (though it would have been more appropriate to have a look at the yaw angle/wind speed relation, but hardly any of such curves are measured).

Figure 8.1 gives the relation between a theoretical and measured rotor speed curve (field measurement) of the CWD 2740 windmill (a windpump with a rotor diameter of 2.74 meters equipped with an side vane and an inclined hinge main vane; for further details see the advanced module of Kragten).

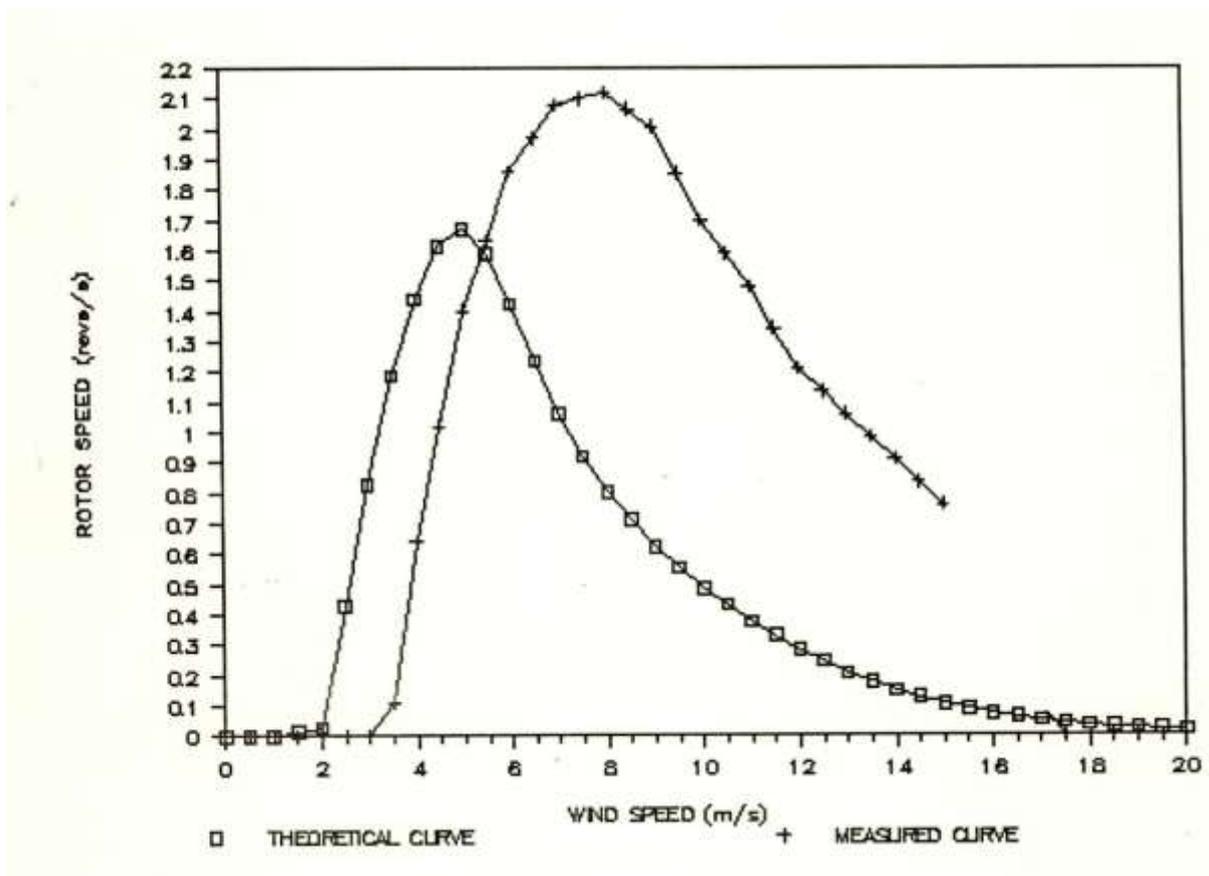


Figure 8.1 Calculated and measured [3] control curve of CWD 2740

The difference between the theoretical and measured curve is considerable.

What are the reasons? The theory is based on many assumptions, for instance that the main vane is a flat plate in a smooth air stream. This is not the case. The vane is in the wake of the rotor, hence located in a very turbulent area. Theory only partly considers this fact. Some influences are not incorporated in the theory, for example the friction in the head bearings. Measurements in the past have shown significant friction. Assumptions are made with regard to the so-called 'self-orientating moment', i.e. the moment generated by the rotor (still partly understood) and the side force.

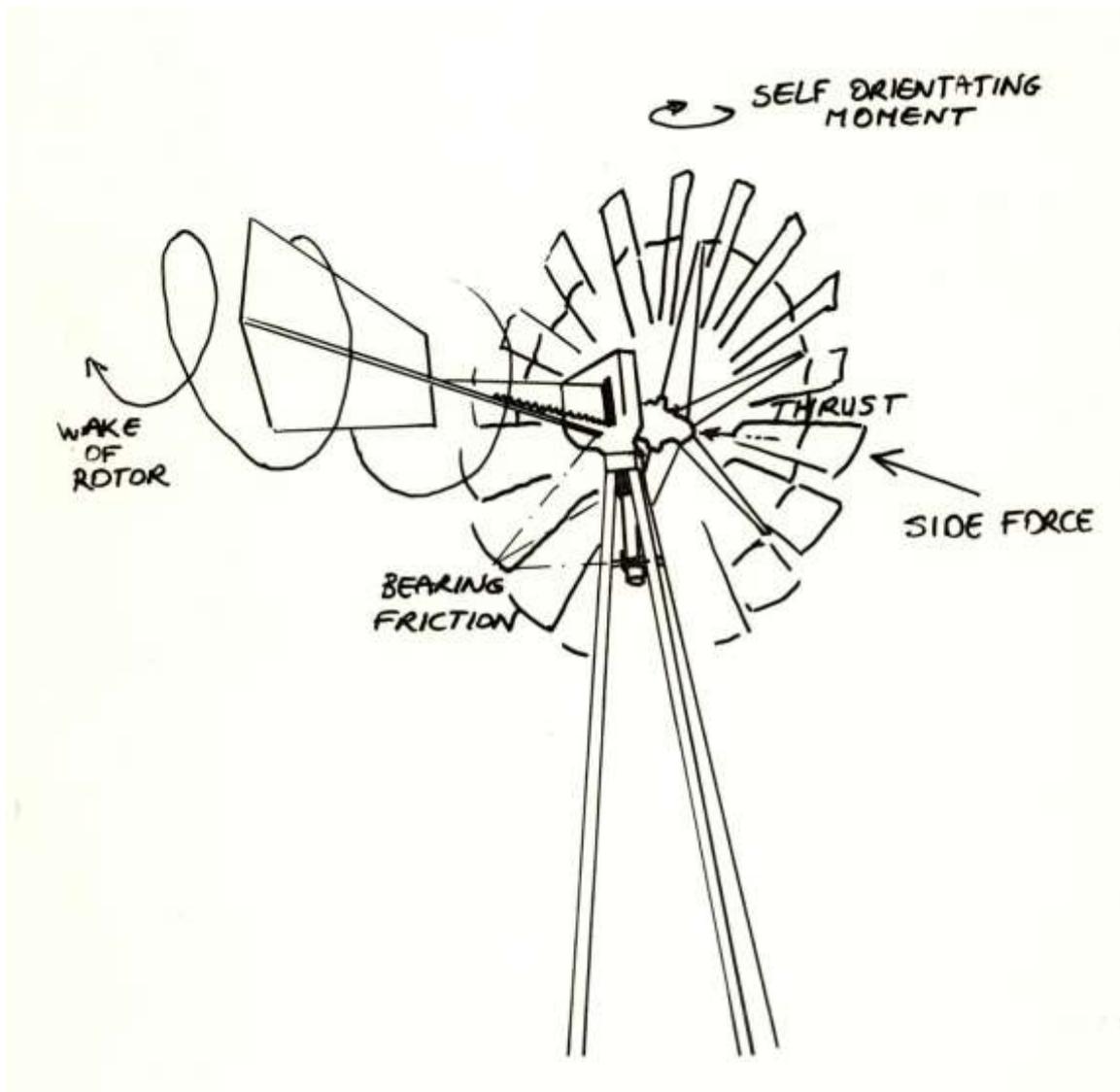


Figure 8.2 Some factors that cause differences between theory and practice

It may be concluded that there is still room for improvement on the theory of control and safety of water pumping windmills.

9 CONCLUSIONS

Reliable and simple automatic control and safety systems exist for water pumping windmills.

Though the actual control characteristics deviate considerably from the ideal characteristics this does not lead to dramatic losses of power.

There is still room for improvement for the theory on the control and safety of water pumping windmills.

REFERENCES

- [1] T. Lindsay Baker, A field guide to American windmills,
University of Oklahoma Press, 1985, ISBN 0-8061-1901-2.
- [2] K. Kieft et al, Windtunnel measurements on the control behaviour of a scale model of the CWD 5000, CWD-UT report WM-083, july 1984.
- [3] H. Oldenkamp, Rotor and yaw measurements on CWD water pumping windmills, OKE services, january 1990.
- [4] Smulders, P.T., Schermerhorn, R.E.
The design of fast running water pumping windmills
Proceedings of the conference of AWEA,
Pasadena, USA, September 1984
- [5] Smulders, P.T. Kragten, A., et al
Innovative control and safety systems for water pumpers
EWEC 84, Hamburg, pp 746-754, paper N5.
- [6] Smulders, P.T., Logtenberg, A.
Control and safety systems for wind pumpers, in particular the hinged side vane mechanism
Proceedings International conference on solar and wind energy applications
1985, Beiing, China, Proceedings pp 21-30
- [7] Smulders, P.T., Cleijne, J.W., Cong Xianzi, Wang Xibo (CARD C)
CWD 2000 wind pump tested full scale in CARD C 12x16 wind tunnel
Proceedings EWEC 89, Glasgow, UK
Peter Pergrinus Ltd, London, UK, pp.736-740
- [8] Lysen, E.H.
Introduction to wind energy
CWD 82-1 publication
May 1983

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ANNEX A CONTROL THEORY

The theory is explained in detail in the advanced module on safety systems in this course by Kragten. This annex gives the base line of the theory, it may be used as a guide when studying the advanced course.

It appears that the theory to predict the static control behaviour consists of 4 basic equations.

First equation

The first equation is the equilibrium equation; the balance of moments. With:

M_{in} = the moment turning the rotor into the wind

M_{out} = the moment turning the rotor out of the wind

the first equation reads:

$$M_{in} = M_{out} \quad (1)$$

The balance of moments is illustrated in figure A.1 in case of a side vane and inclined hinge main vane.

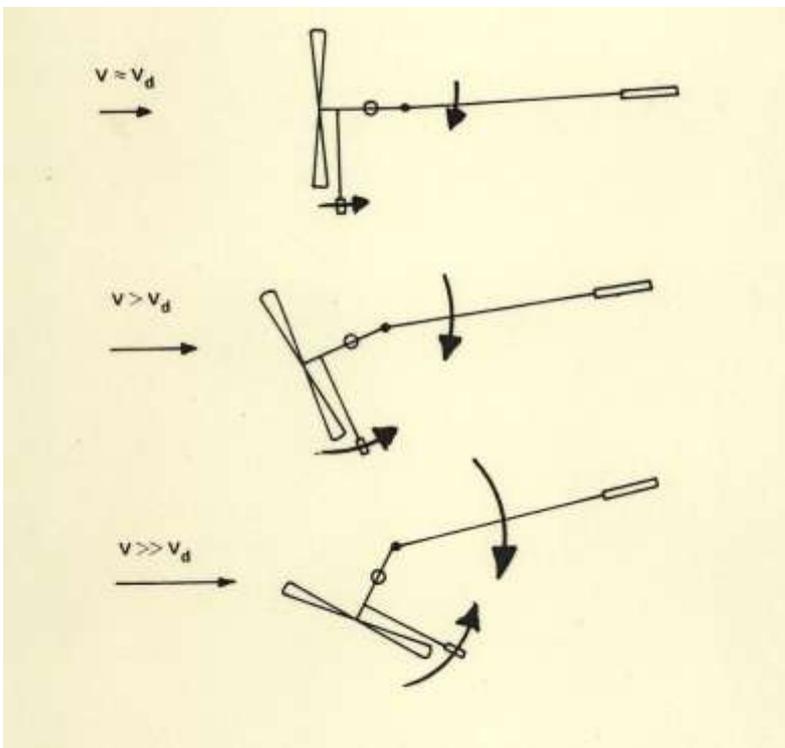


Figure A.1 Balance of moments for several wind speeds

The other 3 equations give a further description of the two balancing moments; one equation for M_{out} , the other two for M_{in}).

Second equation

The second equation is a further breakdown of M_{out} into basic parameters. Let us take the side vane situation as example (a similar story holds for the second option; eccentric rotor). We will have a look in what way the moment depends on other variables. Note that the side vane is a flat plate in an air stream. The formula of the force F on a flat plate in an air stream reads:

$$F = C_N \frac{1}{2} \rho A V^3$$

where C_N : normal coefficient, depending on the angle of attack δ (= yaw angle) and the shape of the plate
 ρ : air density
 A : area of flat plate
 V : wind speed

In shorthand the formula reads:

$$F = f(C_N, \rho, A, V)$$

where $f(..)$ means 'function of', in other words 'depends on'.

With $C_N = f(\delta, \text{geometry})$

the force F will be;

$$F = f(\delta, V, \text{geo})$$

Note that ρ is left out. This is because the air density is not a real variable, it hardly changes for a chosen windmill site.

The wind force F on the plate will exert a moment M_{out} on the head.

$$M_{out} = F \cdot R_v$$

where R_v is the distance from the side vane to the tower axis.

Rewriting the formula in shorthand;

$$M_{out} = f(\delta, V, \text{geo}) \tag{2}$$

as the R_v is a geometrical variable.

Summarizing we find that the moment of the side vane trying to turn the rotor out of the wind depends on;

- 1-yaw angle δ
- 2-wind speed V
- 3-the geometry of the mill (shape of vane, size of vane, distance R_v)

This is the second equation.

Remember that only the base line is highlighted in this annex, the details of formula (2) can be found in the advanced module of Kragten for several windmill configurations. An example of how M depends on δ and V for a specific wind pump design is given in figure A.2 (from Kragten).

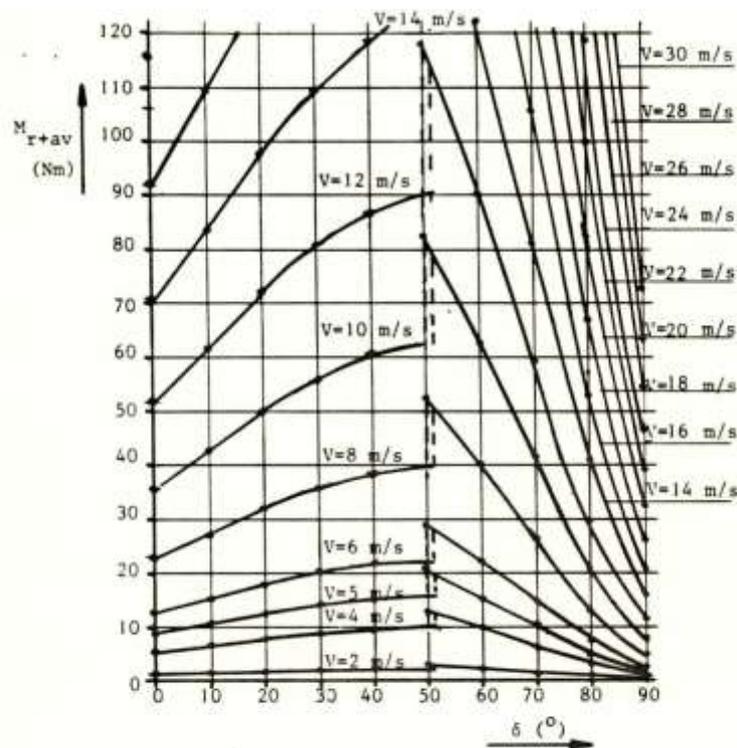


Figure A.2 The M_{out} as function of δ , V for CWD 2740 geometry

The third equation

The last two equations deal with the moment M_{in} that tries to turn the rotor into the wind.

The third equation regards the vane as a flat plate in the wind, just like the side vane. This will result in a formula that resembles formula 2, with the side vane in the wind. In fact the formula reads;

$$M_{in} = f(\alpha_{mv}, V, \text{geo}, a)$$

where α_{mv} = the angle of attack of the main vane. Note the similarity, in both cases it is the angle of attack.
 a = factor to account for the fact that the vane is in the wake of the rotor and that it will therefore experience a smaller wind speed than the undisturbed speed.

As the factor 'a' is generally taken as 0.5, this factor can further be regarded as a constant. Thus formula 3 reads;

$$M_{in} = f(\alpha_{mv}, V, \text{geo}) \quad (3)$$

The fourth equation

The formula describes the equilibrium of the vane. There are two moments on the vane; a moment due to the wind pressure, which (practically) equals M_{in} , and a moment caused by the spring (ecliptic control) or weight (hinged side vane or inclined hinge main vane).

Shortly the equilibrium reads, e.g. for an ecliptic control;

$$M_{in} = M_{spring}$$

M_{spring} depends on the position of the vane relative to the head (as this determines the elongation of the spring) and the mill geometry:

$$M_{\text{spring}} = f(\text{position, geo})$$

The position of the vane relative to the head can be described by the difference between the yaw angle δ and the angle of attack α_{mv} . This leads to:

$$M_{\text{spring}} = f(\delta, \alpha_{\text{mv}}, \text{geo})$$

Substituting this in the equilibrium formula gives the fourth equation:

$$M_{\text{in}} = f(\delta, \alpha_{\text{mv}}, \text{geo}) \tag{4}$$

Overview of results

The results are displayed in table A.1.

table A.1 Formulas describing control behaviour

BACKGROUND	FORMULA	Nr
Balance of moments	$M_{in} = M_{out}$	1
Side vane in air stream	$M_{out} = f(\delta, V, \text{geo})$	2
Main vane in air stream	$M_{in} = f(\alpha_{mv}, V, \text{geo})$	3
Equilibrium of main vane	$M_{in} = f(\delta, \alpha_{mv}, \text{geo})$	4

Figure A.3 displays the variables.

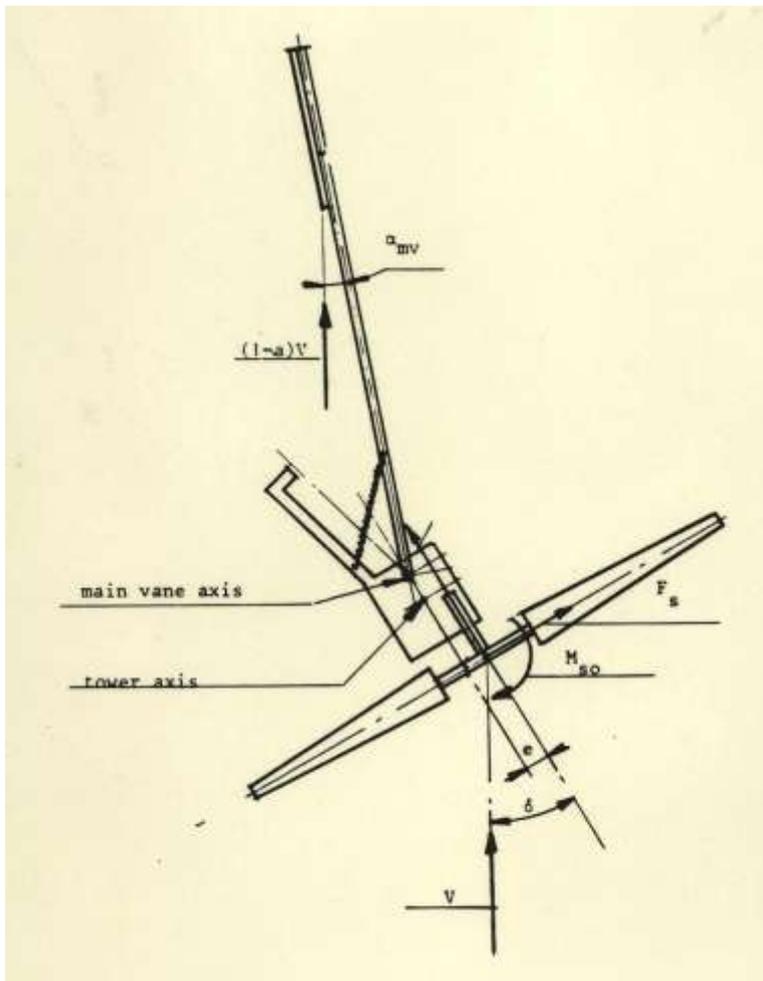


Figure A.3 Illustration of variables in table A.1

Establishing the control behaviour

The basic equations are now found. Now we can establish the control behaviour for a given windmill: the relation between the yaw angle and the wind speed. For a given windmill the variable 'geo' will be known. The set of 4 equations will then reduce to:

$$M_{\text{out}} = M_{\text{in}} \quad (5)$$

$$M_{\text{out}} = f(\delta, \alpha_{\text{mv}}) \quad (6)$$

$$M_{\text{in}} = f(\alpha_{\text{mv}}, V) \quad (7)$$

$$M_{\text{in}} = f(\alpha_{\text{mv}}, \delta) \quad (8)$$

We now have 4 equations with 5 variables, i.e. M_{out} , M_{in} , δ , α_{mv} and V . Choosing a certain wind speed V leaves 4 equations with 4 variables. This results in a value for the other 4 variables, a.o. the yaw angle δ .

For each wind speed V we find the corresponding yaw angle δ . This means that the control characteristic, i.e. the δ - V curve, can be established. Kragten has worked this out for several windmills.

Two notes to be complete:

1-Kragten also incorporates the so-called self-orienting moment of the rotor (indicated with the subscript r) in the control behaviour. For instance for the CWD 2740 windmill the moment $M_{r+\text{av}}$ is presented (advanced module, figure 29). The self-orienting moment is however small in comparison with the other moments. It is therefore not presented in the basic theory in this annex.

2-Kragten derives the instantaneous curves, no averages over longer periods. For instance for the CWD 2740 figure 38 in the advanced module gives the relation between the rotor speed and wind speed for a given *point in time*. Field measurements generally give 10 minutes *averages*. Within 10 minutes the wind speed varied, and the rotor speed with it. The instantaneous curve of the CWD 2740 has therefore been modified to figure 8.1 to account for the variations in the 10 minutes time span (with some assumptions).